Conditional Mean-Variance and Mean-Semivariance models in portfolio optimization

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Abstract

It is known that the historical observed returns used to estimate the expected return provide poor guides to predict the future returns. Consequently, the optimal portfolio weights are extremely sensitive to the return assumptions used. Getting information about the future evolution of different asset returns, could help the investors to obtain more efficient portfolio. The solution will be reached by estimating the portfolio risk by conditional variance or conditional semivariance. This strategy allows us to take advantage of returns prediction which will be obtained by nonparametric univariate methods. Prediction step uses kernel estimation of conditional mean. Application on the Chinese and the American markets are presented and discussed.

Keywords: Conditional Semivariance, Conditional Variance, DownSide Risk, Kernel Method, Nonparametric Mean prediction.

1. Introduction

Investment strategies and their profitability have always been a hot topic for people with an interest in financial assets. The modern asset allocation theory was originated from the Mean-Variance portfolio model introduced by Markowitz (1952), see also Markowitz (1959) and Markowitz (1990). The original Markowitz model simply dealt with the use of historical returns. Variance is commonly used as a risk measure in portfolio optimisation to find the trade-off between the risk and return. In practice, the expected returns and variances are calculated using historical data observed before the portfolio optimization date and are used as proxies for future returns.This practice is not purely guesswork, there are well-developed nonparametric approaches to obtain good forecasts.

Markowitz's portfolio optimization requires the knowledge of both the expected return and the covariance matrix of the assets. It is well known that the optimum portfolio weights are very sensitive to return expectations which are very difficult to determine. For instance, historical returns are bad predictors of the future returns if we use the classical arithmetic mean (Michaud, 1989; Black and Litterman, 1992; Siegel and Woodgate, 2007). Estimating covariance matrices is a delicate statistical challenge that requires sophisticated methods (see Ledoit and Wolf (2004)). It is fair to state that, due to the large statistical errors of the input of Markowitz's portfolio optimization, its result is not reliable and should be considered very cautiously. This led Levy and Roll (2010) to turn the usual approach up-side-down and found that minor adjustments of the input parameter are much needed, well within the statistical uncertainties.

There are various models to used in forecasting future financial time series. For example, Delatola and Griffin (2011) developed a prediction method based on Bayesian nonparametric modelling of the return distribution with stochastic volatility and Kresta (2015) proposed to use of a GARCH-copula model in Portfolio Optimization .

In this paper, we propose a radical different methods by including informations about the possible future returns in the estimation step obtained by kernel nonparametric prediction. Then, we can improve the quality of portfolio optimisation.

Firstly, we exhibit the classical Markowitz model developed in 1952.

Let us say that there are m assets to constitute a portfolio P and denote by r_{jt} the return of asset j on date t, $t = 1, \dots, N$, and M the estimated variance-covariance matrix of the returns $(r_1, \dots, r_m)$,

$$M = \frac{1}{N} \sum_{t=1}^{N} \begin{bmatrix} (r_{1t} - \bar{r}_1)^2 & (r_{1t} - \bar{r}_1)(r_{2t} - \bar{r}_2) & (r_{1t} - \bar{r}_1)(r_{mt} - \bar{r}_m) \\ (r_{2t} - \bar{r}_2)(r_{1t} - \bar{r}_1) & (r_{2t} - \bar{r}_2)^2 & (r_{2t} - \bar{r}_2)(r_{mt} - \bar{r}_m) \\ \vdots & \vdots & \vdots \\ (r_{mt} - \bar{r}_m)(r_{1t} - \bar{r}_1) & (r_{mt} - \bar{r}_n)(r_{2t} - \bar{r}_2) & (r_{mt} - \bar{r}_m)^2 \end{bmatrix}. \quad (1)$$

The optimization program is then the following

$$\min_{\omega} \omega^\top M \omega, \text{ subject to } \omega^\top \mu = E^*, \omega^\top \mathbf{1} = 1, \quad (2)$$

where $\omega^\top = (\omega_1, \dots, \omega_m)$ is the portfolio vector weight, $\mu^\top = (\bar{r}_1, \dots, \bar{r}_m) = (\frac{1}{N}\sum_{t=1}^{N} r_{1t}, \dots, \frac{1}{N}\sum_{t=1}^{N} r_{mt})$ the empirical mean returns and E^* is a target expected portfolio return.

Using Lagrangian method, the explicit solution of solution of (2) is :

$$\omega_* = \frac{\alpha E^* - \lambda}{\alpha\theta - \lambda^2} M^{-1}\mu + \frac{\theta - \lambda E^*}{\alpha\theta - \lambda^2} M^{-1}\mathbf{1}, \quad (3)$$

where $\alpha = \mathbf{1}^\top M^{-1}\mathbf{1}$, $\lambda = \mu^\top M^{-1}\mathbf{1}$ and $\theta = \mu^\top M^{-1}\mu$.

This model depends strictly on the assumptions that the assets returns follow normal distribution and investor has quadratic utility function. However, these two conditions are not checked. Many researchers have showed that the assets returns distribution are asymmetric and exhibit skewness, see Tobin (1958); Arditti (1971); Chunhachinda et al. (1997); Prakash et al. (2003). These authors have proposed a DownSide Risk (DSR) measures such as Semivariance (SV) and conditional value at risk (CVaR). These DSR measures are consistent with investor's perception towards risk as they focus on return dispersions below any benchmark return B chosen by the investor. Below, we focus only on the Semivariance risk measure which is often considered as a more plausible risk one than the variance. The associated optimization program is the following:

$$\min_{\omega} \omega^\top M_{SR}\omega \text{ subject to } \omega^\top \mu = E^*, \omega^\top \mathbf{1} = 1, \tag{4}$$

where M_{SR} is the matrix with coefficients

$$\Sigma_{ijB} = \frac{1}{N}\sum_{t=1}^{V}(r_{it} - B)(r_{jt} - B)$$

such that V is the period in which the portfolio underperforms the target return B.

The major obstacle to get the solution of this problem is that the semicovariance matrix is endogenous (see Estrada (2004, 2008)), that is, the change in weights affects the periods in which the portfolio underperforms the target rate of return, which in turn affects the elements of the Semivariance matrix.

Many authors propose different methods to estimate the elements of M_{SR} in order to resolve problem defined in equation (4). Among them, Hogan and Warren (1974) propose to use the Frank-Wolf algorithm but the main disadvantage of this algorithm is its slow convergence rate. Moreover, during early iteration, this algorithm tends to decrease the objective function. Ang (1975) proposes to linearise the Semivariance so that the optimization problem can be solved using linear programming. However, this method ignores the inter-correlations between securities. Harlow (1991) also considers problem (4) and generates Mean-Semivariance efficient frontier, which he compares to the Mean-Variance efficient frontier. Mamoghli and Daboussi (2008) improve Harlow approach. Their model permits to surmount the problem of inequality of the Semicovariance measures which occurs in the Mean-Semivariance model of Harlow. Markowitz et al. (1993) transform the Mean-Semivariance problem into a quadratic problem by adding fictitious securities. Estrada (2008) proposes a simple and accurate heuristic approach that yields a symmetric and exogenous Semicovariance matrix, which enables the determination of Mean-Semivariance optimal portfolios by using the well-known closed-form solutions of Mean-Variance problems. In Athayde (2001) and (Athayde, 2003), there is an iterative algorithm which is used to construct a Mean-DownSide Risk portfolio frontier. This algorithm is improved by Ben Salah et al. (2016a) and Ben Salah et al. (2016b) by introducing nonparametric estimation of the returns in order to get a better smooth efficient frontier.

A knowledgeable investor should have an overview on market development. For a fixed amount to invest now, he has to predict, day by day (or month by month, ...), the optimal return that his investment could bring to him.

In this paper, we will develop a rule of decision to optimize the portfolio selection through reducing the risk calculated using the classical variance. A new approach, called conditional Markowitz optimization method, will be used to determine an optimal portfolio. This portfolio is obtained by minimizing the so-called *conditional risk*. This risk may in turn be broken down into two versions: Mean-Variance and Mean-Semivariance approaches. The main idea is to anticipate the values of the assets returns on the date $N+1$ (knowing the past) and incorporate this information in the optimization model.

The rest of the article is organized as follows. Section 2 introduces the nonparametric conditional risk. We start by giving an overview on nonparametric regression and prediction, as well as conditional variance and covariance definitions. The conditional Mean-Variance and the conditional Mean-Semivariance models are exhibited in this Section. Numerical studies based on the Chinese and the American Market dataset are presented in Section 3. The last Section is devoted to conclusion and further development.

2. Nonparametric Conditional Risk

Here, we start by giving some general concepts and results concerning nonparametric regression and prediction. Then, we define the Conditional Mean-Variance and Mean-Semivariance models and we exhibit the corresponding algorithms to get the optimal portfolio.

2.1. Nonparametric Regression Model

In the following subsection, we outline the mechanics of kernel regression estimation.

Let (X, Y) be a pair of explanatory and response random variables with joint density $f(x, y)$, and $)(Y_i, X_i)_{i=1,T}$ T-copies of (X, Y). We suppose that X is m-dimensional. Using a quadratic criterion, the best prediction of Y based on $X = x$ is the conditional expectation $E(Y \mid X = x) = g(x)$.

Using $(Y_i, X_i)_{i=1,T}$, we will focus on the estimation of the unknown mean response $g(x)$.

The regression model is

$$Y_t = g(x_t) + \epsilon_t, \tag{5}$$

where $g(.)$ is unknown. The errors ϵ_t satisfy

$$E(\epsilon_t) = 0, \qquad V(\epsilon_t) = \sigma_\epsilon^2, \qquad Cov(\epsilon_i, \epsilon_j) = 0 \text{ for } i \neq j. \tag{6}$$

To derive the estimator note that we can express $g(x)$ in terms of the joint probability density function $f(x, y)$ as follows:

$$g(x) = E(Y \mid X = x) = \int y f(y \mid x) dy = \frac{\int y f(x, y) dy}{\int f(x, y) dy} \tag{7}$$

Using kernel density estimation of the joint distribution and the marginal one (see (Bosq and Lecoutre, 1987)), the Nadaraya-Watson estimator for $g(x)$ is given by :

$$\hat{g}(x) = \frac{\sum_{t=1}^{T} Y_t \mathcal{K}_H(X_t - x)}{\sum_{t=1}^{T} \mathcal{K}_H(X_t - x)} \tag{8}$$

where $\mathcal{K}_H(x) = |H|^{-1/2}\mathcal{K}(|H|^{-1/2}x)$ with H is the $m \times m$ matrix of smoothing parameters which is symmetric and positive definite and $|H|$ its determinant. The function $\mathcal{K} : \mathbb{R}^m \to [0, \infty)$ is a probability density.

By the way, the conditional variance is given by

$$\sigma^2(x) = E((Y - g(x))^2 \mid X = x), \tag{9}$$

and its kernel estimator is given by

$$\hat{\sigma}^2(x) = \frac{\sum_{t=1}^{T} (Y_t - g(\hat{x}))^2 \mathcal{K}_H(X_t - x)}{\sum_{t=1}^{T} \mathcal{K}_H(X_t - x)}. \tag{10}$$

If $Y = (Y_1, \ldots, Y_m)$ is a random vector, the conditional covariance between Y_1 and Y_2 given $X = x$ is defined as

$$\sigma_{12}(x) = E(Y_1 - g_1(x))(Y_2 - g_2(x)) \mid X = x), \tag{11}$$

and its kernel estimator is given by

$$\hat{\sigma}_{12}(x) = \frac{\sum_{t=1}^{T} (Y_{1t} - \hat{g}_1(x))(Y_{2t} - \hat{g}_2(x)) \mathcal{K}_H(X_t - x)}{\sum_{t=1}^{T} \mathcal{K}_H(X_t - x)}, \tag{12}$$

where $g_1(x)$ and $\hat{g}_1(x)$ (respectively $g_2(x)$ and $\hat{g}_2(x)$) are the $E(Y_1 \mid X = x)$ and its kernel estimator (respectively the $E(Y_1 \mid X = x)$ and its kernel estimator) defined above.

The choice of the bandwidth matrix H is the most important factor affecting the accuracy of the estimator (8), since it controls the orientation and amount of smoothing induced.

The simplest choice for this matrix is to take $H = h\mathbb{I}_m$, where h is a unidimensional smoothing parameter and $\mathbb{I}_m$ is the $m \times m$ identity matrix. Then we have the same amount of smoothing applied in all coordinate directions and the kernel estimator (8) has the form

$$\hat{g}(x) = \frac{\sum_{t=1}^{T} Y_t \mathcal{K}(\frac{X_t - x}{h})}{\sum_{t=1}^{T} \mathcal{K}(\frac{X_t - x}{h})} = \frac{\sum_{t=1}^{T} Y_t \mathcal{K}_h(X_t - x)}{\sum_{t=1}^{T} \mathcal{K}_h(X_t - x)}, \tag{13}$$

where $\mathcal{K}_h(x) = h^{-m}\mathcal{K}(\frac{x}{h})$. The estimators (10) and (12) are derived similarly.

Another choice, relatively easy-to-manage, is to take the bandwidth matrix equal to a diagonal matrix, which allows for different amounts of smoothing in each of the coordinates.

The following kernel functions are used most often in multivariate nonparametric estimation:

- product kernel: $\mathcal{K}(x) = K(x_1) \times \cdots \times K(x_m) = \prod_{j=1}^{m} k(x_j)$ where k(.) is an univariate density function.
- symmetric kernel: $C_{k,m} k(\|x\|_2^{1/2})$.

The multivariate Gaussian density $K(x) = (2\pi)^{-m/2} exp(\frac{x^\top x}{2})$ is a product and symmetric kernel.

In the rest of this paper, we will consider only the product kernel and the bandwidth matrix $H = diag(h_1, \ldots, h_m)$. Then, for $x = (x_1, \ldots, x_m)$, the estimator (8) should be written as follows:

$$\hat{g}(x) = \frac{\sum_{t=1}^{T} Y_t k(\frac{X_{1t}-x_1}{h_1}) \times \cdots \times k(\frac{X_{mt}-x_m}{h_m})}{\sum_{t=1}^{T} k(\frac{X_{1t}-x_1}{h_1}) \times \cdots \times k(\frac{X_{mt}-x_m}{h_m})}, \tag{14}$$

with $h_j = T^{-1/m+4} \hat{\sigma}_j$ where $\hat{\sigma}_j$ is an estimator of the standard deviation of the random variable X_j. These bandwidths are optimal using the minimisation of the Asymmetrical Mean Integrated Error criterion (see for example (Wasserman, 2006)).

The estimators (10) and (12) are derived similarly.

2.2. Nonparametric Prediction

Nonparametric smoothing techniques can be applied beyond the estimation of the autoregression function. Consider a m-multivariate stationary time series $\{(r_{1t}, \ldots, r_{mt}),\ t = 1, \ldots, N\}$. We consider the processes (X_t, Y_t) defined as follows

$$X_t = (r_{1t}, \ldots, r_{mt}) \qquad\qquad Y_t = r_{j(t+1)}, \tag{15}$$

and we are interested in predicting the return of a given asset j on time $N+1$. This problem is equivalent to the estimation of the regression $g(.)$ function presented above. Then

$$g(r_{1N},\ldots,r_{mN}) = \mathbb{E}(Y_N \mid X_N = x_N) = \mathbb{E}(r_{j(N+1)} \mid x_N = (r_{1N},\ldots,r_{mN})). \tag{16}$$

Let $T = N-1$. Using the kernel method, we get easily the following estimators for the conditional expectation, variance and covariance:

- **Conditional expectation:**

$$\bar{r}_{c,j} = \hat{g}(r_{1N},\ldots,=r_{mN}) = \frac{\sum_{t=1}^{T} r_{j(t+1)} K(\frac{r_{1t}-r_{1N}}{h}) \times \cdots \times K(\frac{r_{mt}-r_{mN}}{h})}{\sum_{t=1}^{T} K(\frac{r_{1t}-r_{1N}}{h}) \times \cdots \times K(\frac{r_{mt}-r_{mN}}{h})}, \tag{17}$$

- **Conditional variance :**

$$\hat{\sigma}^2_{c,j}(r_{1N},\ldots,r_{mN}) = \frac{\sum_{t=1}^{T} (r_{j(t+1)} - \bar{r}_{c,j})^2 K(\frac{r_{1t}-r_{1N}}{h}) \times \cdots \times K(\frac{r_{mt}-r_{mN}}{h})}{\sum_{t=1}^{T} K(\frac{r_{1t}-r_{1N}}{h}) \times \cdots \times K(\frac{r_{mt}-r_{mN}}{h})}, \tag{18}$$

- **Conditional covariance :**

$$\hat{\sigma}_{c,ij}(r_{1N},\ldots,r_{mN}) = \frac{\sum_{t=1}^{T} (r_{i(t+1)} - \bar{r}_{c,i})(r_{j(t+1)} - \bar{r}_{c,j}) K(\frac{r_{1t}-r_{1N}}{h}) \times \cdots \times K(\frac{r_{mt}-r_{mN}}{h})}{\sum_{t=1}^{T} K(\frac{r_{1t}-r_{1N}}{h}) \times \cdots \times K(\frac{r_{mt}-r_{mN}}{h})}, \tag{19}$$

Comments

1. To estimate conditional expectation, variance and covariance, we supposed that the future depends on the immediate past. This hypothesis, considered as 1-Markovian condition, is natural because the future returns are strongly correlated to the recent past.
2. In Statistics literature, many other multivariate kernels are proposed like the Spherical/radial-symmetric kernel or the multivariate Epanechnikov (spherical) one.

3. Nonparametric methods are typically indexed by a bandwidth or tuning parameter which controls the degree of complexity. The choice of bandwidth is often critical to implementation: under- or over-smoothing can substantially reduce precision. The standard approach to the bandwidth problem is to choose a bandwidth that minimizes some measures of global risk for the entire regression function, usually Mean Integrated Squared Error (MISE), i.e. the expected squared error integrated over the entire curve. The optimal bandwidth is then estimated either using plug-in estimators of the minimizer of the asymptotic approximation to MISE or using an unbiased data-based estimator of the MISE . This is the cross-validation method. This method was analyzed in Sarda (1993). It is recommended by (Altman and Leger, 1995) when large samples are available. As to the estimation of the conditional covariance matrix, one may use different bandwidths for different elements of this matrix. However, the resulting estimation with different bandwidths cannot be guaranteed to be positive definite (Li et al. (2007)). In practice, the positive definiteness is a desirable property. Thus, we suggest using the same bandwidth for all elements.

2.3. Conditional Mean-Variance Model

Our goal here is to constitute an optimal portfolio using conditional criterion. In our opinion, it is natural to use conditional information to provide a timely and effective solution.

We suppose that they are, as in the previous section, m assets to be used for constructing a well diversified portfolio. Optimizing asset allocation is simply defined as the process of mixing asset weights of a portfolio within the constraints of an investor's capital resources to yield the most favourable risk-return trade-off. The risk here is defined by the conditional variance of ,the portfolio return.

Let $\omega = (\omega_1, \ldots, \omega_m)^\top$ be the portfolio weight vector and $r_{pt} = \omega_1 r_{1t} + \omega_2 r_{2t} + ... + \omega_m r_{mt}, t = 1 \ldots, N$, N-realisations of the the portfolio return R_p.

Using conditional mean-variance criterion, the optimization model is

$$\min_{\omega} \omega^\top M_c \omega, \text{ subject to } \omega^\top \mu_c = E^*, \omega^\top \mathbf{1} = 1, \tag{20}$$

where $\mu_c = (\bar{r}_{c,1}, \ldots, \bar{r}_{c,m})^\top$, E^* is a given target return and M_c is the conditional covariance matrix whose elements are $\hat{\sigma}_{c,ij}(r_{1N}, \ldots, r_{mN})$.

Using Lagrangian method, an explicit solution of (20) is given by:

$$\omega_{c,*} = \frac{\alpha_c E^* - \lambda_c}{\alpha_c \theta_c - \lambda_c^2} M_c^{-1} \mu_c + \frac{\theta_c - \lambda_c E^*}{\alpha_c \theta_c - \lambda_c^2} M_c^{-1} \mathbf{1}, \tag{21}$$

where $\alpha_c = \mathbf{1}^\top M_c^{-1} \mathbf{1}$, $\lambda_c = \mu_c^\top M_c^{-1} \mathbf{1}$ and $\theta_c = \mu_c^\top M_c^{-1} \mu_c$.

Then the conditional risk, CR, is given by

$$CR = (\omega_{c,*}^\top M_c \omega_{c,*}). \tag{22}$$

2.4. Conditional Mean-Semivariance Model

Let us recall that the main criticism to variance, used by Markowitz (1952) as measure of risk is, in essence, that it gives the same importance and the same weight to gains and losses, also the use of variance suppose that returns are normally distributed . That is why Markowitz (1959) argues for another more plausible measure of risk that he calls the Semivariance which takes into consideration the asymmetry and the risk perception of investors. For motivations, details and theoretical result on this method, see for example Vasant et al. (2014) .

Let B the benchmark return(a threshold which captured the risk perspectives from investors to investors). It is a target return which can be equal to 0. The conditional mean-Semivariance model is the following:

$$\min_{\omega} \omega^\top M_{\{c,SR\}} \omega \text{ subject to } \omega^\top \mu_c = E^*, \omega^\top \mathbf{1} = 1, \tag{23}$$

where $M_{\{c,SR\}}$ is the matrix with coefficients

$$\hat{\sigma}_{ijB} = \frac{\sum_{t=1}^{V}(r_{it}-B)(r_{jt}-B)K(\frac{r_{1t}-r_{1N}}{h})\times\cdots\times K(\frac{r_{mt}-r_{mN}}{h})}{\sum_{t=1}^{V}K(\frac{r_{1t}-r_{1N}}{h})\times\cdots\times K(\frac{r_{mt}-r_{mN}}{h})}, \qquad (24)$$

such that V is the period in which the portfolio underperforms the target return B.

Remark. The coefficients $\hat{\sigma}_{ijB}$ are computed differently of those from (19). To do it in the same way, we should re-index the observations (such that portfolio under performs the target return B) in order to get a new time series process and then apply (19). This modification is not very helpful given the abundance of data.

Resolving this problem is not easy because the matrix $M_{\{c,SR\}}$ is endogenous. Based on Athayde algorithm (see (Athayde, 2001)), we develop, in the following, an iterative algorithm that could resolve the optimization problem without enormous difficulties. The principle is the use of Lagrangian method at each step.

- **Step 0:**
  - Start with $\omega_{c,0} = (\omega^1_{c,0}, \ldots, \omega^m_{c,0})$,
  - compute $r^0_{pt} = \omega^1_{c,0} r_{1t} + \cdots + \omega^m_{c,0} r_{mt}$, $t = 1, \ldots, N$,
  - select the set S_0 of time indices portfolio return observations in which this portfolio $\omega_{c,0}$ had negative deviations i.e. $r^0_{pt} - B \leq 0$.

- Construct the following positive $(m \times m)$ semi-definite matrix $M_{\{c,SR,0\}}$:

$$M_{\{c,SR,0\}} = \frac{1}{\sum_{t \in S_0'} K(\frac{r_{1(t-1)} - r_{1N}}{h}) \times \cdots \times K(\frac{r_{m(t-1)} - r_{mN}}{h})} \times$$

$$\sum_{t \in S_0'} \begin{bmatrix} \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & (r_{it} - B)(r_{jt} - B) K(\frac{r_{1(t-1)} - r_{1N}}{h}) \times \cdots \times K(\frac{r_{m(t-1)} - r_{mN}}{h}) & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots \end{bmatrix}, \tag{25}$$

  where $S_0' = S_0$ or $S_0 = S_0 \setminus \{1\}$ (if $1 \in S_0$).

- **Step 1:** find the portfolio $\omega_{c,1}$ that solves the following problem:

$$\min_{\omega} \omega^\top M_{\{c,SR,0\}} \omega \text{ subject to } \omega^\top \mu_c = E^*, \omega^\top \mathbf{1} = 1. \tag{26}$$

  Using *Lagrangian Method* , the solution to the problem (26) will be given by:

$$\omega_{c,1} = \frac{\alpha_{c,1} E^* - \lambda_{c,1}}{\alpha_{c,1}\theta_{c,1} - \lambda_{c,1}^2} (M_{\{c,SR,0\}})^{-1} \mu_c + \frac{\theta_{c,1} - \lambda_{c,1} E^*}{\alpha_c \theta_{c,1} - \lambda_{c,1}^2} (M_{\{c,SR,0\}})^{-1} \mathbf{1}, \tag{27}$$

  where $\alpha_{c,1} = \mathbf{1}^\top (M_{\{c,SR,0\}})^{-1} \mathbf{1}$, $\lambda_{c,1} = \mu_{c,1}^\top (M_{\{c,SR,0\}})^{-1} \mathbf{1}$ and $\theta_{c,1} = \mu_c^\top (M_{\{c,SR,0\}})^{-1} \mu_c$.

- **Step 2:**
  - compute $r_{pt}^1 = \omega_{c,1}^1 r_{1t} + \cdots + \omega_{c,1}^m r_{mt}$, $t = 1, \ldots, N$,
  - select the set S_1 of index observations in which this portfolio $\omega_{c,1}$ had negative deviations i.e. $r_{pt}^1 - B \leq 0$,

– construct the following positive $(m \times m)$ semi-definite matrix $M_{\{c,SR,1\}}$:

$$M_{\{c,SR,1\}} = \frac{1}{\sum_{t \in S_1'} K(\frac{r_{1(t-1)} - r_{1N}}{h}) \times \cdots \times K(\frac{r_{m(t-1)} - r_{mN}}{h})} \times$$
$$\sum_{t \in S_1} \begin{bmatrix} \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & (r_{it} - B)(r_{jt} - B) K(\frac{r_{1(t-1)} - r_{1N}}{h}) \times \cdots \times K(\frac{r_{m(t-1)} - r_{mN}}{h}) & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots \end{bmatrix}, \tag{28}$$

where $S_1' = S_1$ or $S_1' = S_1 \setminus \{1\}$ (if $1 \in S_1$).

– find the portfolio $\omega_{c,2}$ that solves the following problem:

$$\min_{\omega} \omega^\top M_{\{c,SR,1\}} \omega \text{ subject to } \omega^\top \mu_c = E^*, \omega^\top \mathbf{1} = 1. \tag{29}$$

Using *Lagrangian Method* , the solution to the problem (29) will be given by:

$$\omega_{c,2} = \frac{\alpha_{c,2} E^* - \lambda_{c,2}}{\alpha_{c,2} \theta_{c,2} - \lambda_{c,2}^2} (M_{\{c,SR,1\}})^{-1} \mu_c + \frac{\theta_{c,2} - \lambda_{c,2} E^*}{\alpha_c \theta_{c,2} - \lambda_{c,2}^2} (M_{\{c,SR,1\}})^{-1} \mathbf{1}. \tag{30}$$

- **Step 3:** iterate the previous process to construct a sequence of matrices $M_{\{c,SR,l\}}$ until getting the first matrix $M_{\{c,SR,F\}}$ satisfying the criterion $M_{\{c,SR,F\}} = M_{\{c,SR,F+1\}}$. The Optimal portfolio will be given by:

$$\omega_{c,F+1} = \frac{\alpha_{c,F} E^* - \lambda_{c,F}}{\alpha_{c,F} \theta_{c,F} - \lambda_{c,F}^2} (M_{\{c,SR,F\}})^{-1} \mu_c + \frac{\theta_{c,F} - \lambda_{c,F} E^*}{\alpha_c \theta_{c,F} - \lambda_{c,F}^2} (M_{\{c,SR,F\}})^{-1} \mathbf{1}. \tag{31}$$

and the Conditional Semi Risk value , CSR, will be

$$CSR = \omega_{c,F+1}^\top M_{\{c,SR,F\}} \omega_{c,F+1} \tag{32}$$

Remarks.

1. There is a finite number of iterations to get the optimal solution.
2. In the prediction step (to get the an unobservable values of the returns), we treated separately the evolution of each asset. It is possible to make multivariate (or vectorial) prediction and get jointly the unobservable values for all the assets.
3. Short selling is allowed in this model, i.e. the optimal portfolio can have a negative weight for some assets. To forbid short selling, the additional constraint $\omega_j \geq 0 \; for \; j = 1, \ldots, m$ is necessary.

3. Empirical Analysis

In this section, the performance of the conditional mean-variance and the conditional mean-semivariance optimisation methods are investigated. They are compared to Classical Markowitz and DownSide methods. It is supposed that there is no transaction costs, no taxes and that the benchmark $B = 0$.

3.1. Data

A dataset, drawn from Reuters, was used for this analysis. The original Data are a daily stock returns belonged to two markets:

- The Chinese market (emerging market) with 16 assets. They are 897 daily observations (returns) for each asset from November the 14th, 2011, to July 8th, 2015,
- The American market (developed market) with 19 assets. They are 788 daily observations (returns) for each asset from June, 18th, 2012 to July, 8th, 2015.

To compare the efficiency and the performance of the proposed methods, we use the daily values of

- The Hang Seng Index-HSI that aims to capture the leadership of the Hong Kong exchange, and covers approximately 65% of its total market capitalization,

- The S&P 500 Index that tracks 500 large U.S. companies across a wide span of industries and sectors. The stocks in the S&P 500 represent roughly 70 % of all the stocks that are publicly traded.

The assets returns are calculated from stock prices observed on Thomson Reuters Platform as follows:

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}}, \tag{33}$$

with

- p_t: Stock price at date t,

- p_{t-1}: Stock price at date $t-1$

The prices p_t, $t = 1, \ldots, N$; are adjusted for dividends.

The historical statistics of the asset markets are summarised below.

3.2. Historical statistics

In this subsection, historical statistics are proposed. The goal is to check the normality or not of the returns distribution in order to decide which risk measure is more appropriate to determine the optimal portfolio.

3.2.1. The Chinese Market

Let us start by the Chinese Market. Over the past two decades, the Chinese economy and financial markets have undergone a remarkable transformation and have seen significant growth. More specifically, the Chinese equity market has grown from a once very rudimentary and closed market to one of the largest equity markets in the world. Although most of the Chinese equity market still remains in the hands of controlling parties and domestic investors, authorities have made significant progress in opening the market to foreign capital and

Table 1: Chinese historical analysis

	Abbreviation	Min	Mean	Sd	Skewness	Kurtosis	Max
HangSeng	HSI	-5.84	0.03	1.01	-0.22	1.86	3.80
Agricultural.Bk	A.Bk	-9.90	0.06	1.61	0.70	9.93	10.12
Bank.of.China	B.Ch	-10.98	0.09	1.78	0.69	11.31	10.14
Ind.And.Com.Bank	IACB	-9.90	0.04	1.48	0.06	9.18	9.04
Petrochina	Pet	-9.21	0.04	1.68	0.87	12.09	10.04
China.life.insurance	Ch.L.I.	-10.01	0.10	2.40	0.71	4.02	10.04
China.petroleum	Ch.P.	-10.04	0.04	1.88	0.28	6.85	10.04
Bank.of.Com.	Bk.C.	-10.00	0.08	1.97	0.74	8.88	10.10
Citic.securit	Ci.Se	-7.14	0.00	1.81	1.14	6.81	12.95
China.telecom	Ch.T.	-4.85	0.01	1.81	0.48	1.49	9.28
China.pacific	Ch.P.	-9.98	0.08	2.38	0.55	3.51	13.50
Chinarailway	Ch.R.	-10.14	0.19	2.92	2.65	20.68	26.59
Huaneng	Hua	-10.35	0.11	2.35	0.21	5.90	14.83
Greatwall	Gre	-10.00	0.18	2.63	0.19	1.38	10.63
Dong.feng	Do.F.	-6.96	-0.01	2.19	0.26	0.55	8.13
China.nat.buil	Ch.N.B	-8.96	-0.03	2.16	0.45	2.39	11.12
Tsingtao	Tsi	-10.04	0.01	1.79	0.27	6.86	12.79

increasing the tradable float outstanding-meaning the Chinese equity market has the potential to become a top dominant force within global portfolios. Taking all assets together, we observe that

- $-10.98 \leq Min \leq -4.85$
- $3.80 \leq Max \leq 26.59$
- $-0.03 \leq Mean \leq 0.19$
- $1.01 \leq SD \leq 2.92$
- $0.55 \leq Kurtosis \leq 20.68$

Table 2: China Correlation Matrix

	A.Bk	B.Ch	IACB	Pet	C.L.I	Ch.P.	Bk.C.	Ci.Se	Ch.T.	Ch.P.	Ch.R.	Hua	Gre	Do.F.	C.N.B	Tsi
A.Bk	1.00	0.82	0.80	0.57	0.49	0.59	0.76	0.04	0.04	0.25	0.18	0.01	-0.03	0.01	0.03	0.09
B.Ch	0.82	1.00	0.76	0.55	0.49	0.57	0.74	0.03	0.00	0.23	0.23	-0.00	-0.04	-0.01	-0.02	0.05
IACB	0.80	0.76	1.00	0.60	0.45	0.60	0.72	0.02	0.05	0.20	0.16	-0.01	-0.03	0.01	0.05	0.09
Pet	0.57	0.55	0.60	1.00	0.54	0.74	0.52	0.02	0.06	0.23	0.18	0.00	0.00	0.02	0.06	0.09
C.L.I	0.49	0.49	0.45	0.54	1.00	0.45	0.49	-0.01	0.04	0.23	0.23	0.03	-0.02	0.01	0.03	0.09
Ch.P.	0.59	0.57	0.60	0.74	0.45	1.00	0.57	0.09	0.04	0.19	0.18	0.02	-0.00	0.06	0.11	0.06
Bk.C.	0.76	0.74	0.72	0.52	0.49	0.57	1.00	0.03	0.01	0.26	0.19	-0.01	-0.03	0.00	-0.00	0.11
Ci.Se	0.04	0.03	0.02	0.02	-0.01	0.09	0.03	1.00	0.10	0.05	0.03	-0.01	-0.01	0.12	0.13	0.03
Ch.T.	0.04	0.00	0.05	0.06	0.04	0.04	0.01	0.10	1.00	0.01	0.05	-0.01	-0.02	0.12	0.40	-0.03
Ch.P.	0.25	0.23	0.20	0.23	0.23	0.19	0.26	0.05	0.01	1.00	0.15	-0.03	0.01	0.00	0.01	0.13
Ch.R.	0.18	0.23	0.16	0.18	0.23	0.18	0.19	0.03	0.05	0.15	1.00	0.05	0.00	0.01	0.01	0.16
Hua	0.01	-0.00	-0.01	0.00	0.03	0.02	-0.01	-0.01	-0.01	-0.03	0.05	1.00	0.00	0.01	0.06	0.08
Gre	-0.03	-0.04	-0.03	0.00	-0.02	-0.00	-0.03	-0.01	-0.02	0.01	0.00	0.00	1.00	0.02	0.03	0.03
Do.F.	0.01	-0.01	0.01	0.02	0.01	0.06	0.00	0.12	0.12	0.00	0.01	0.01	0.02	1.00	0.12	0.00
C.N.B	0.03	-0.02	0.05	0.06	0.03	0.11	-0.00	0.13	0.40	0.01	0.01	0.06	0.03	0.12	1.00	-0.01
Tsi	0.09	0.05	0.09	0.09	0.09	0.06	0.11	0.03	-0.03	0.13	0.16	0.08	0.03	0.00	-0.01	1.00

- $-0.22 \leq Skwenes \leq 2.65$

It means that the behaviour of assets return is different from an asset to another. However, the returns distribution are more peaked than a Gaussian distribution (the Skewness $\neq 0$). This statement should favour the use of the DSR methods to optimize a portfolio. As a first time, we process the data as they are normally distributed. Our first goal is to compare the efficiency of classical Markowitz Mean-Variance method versus the proposed Conditional-Mean-Variance method. The second one is to compare the Mean-DownSide Risk method to the Conditional one.

To analyse the possible correlation between the different assets, It is helpfull to know the correlation matrix. This matrix is presented in Table (2) It appears that the returns are differently correlated between each other. For example, Agricultural.Bk is strongly correlated to Bank.of.China and Ind.And.Com.Bank assets, and very weakly correlated to Dong.feng and China.nat.buil assets. More generally, the correlation between two stocks is larger when they are from banking sector than when they belong to different industries. The use of Principal Component Analysis (PCA) could reduce the number of assets to constitute the optimal portfolio. This step is omitted in this paper.

Remark.

In this matrix, we have excluded, the correlation between the HangSeng Index and the other assets because we will use it as a benchmark to compare its return to the optimal portfolio's.

3.2.2. The Americain Market

As for the Chinese Market, similar historical statistics are done for the American one. In the beginning, we display and comment the descriptive statistics, then we exhibit and comment the correlation matrix.

Taking all assets together, we observe that

- $-14.34 \leq Min \leq -0.90$
- $2.54 \leq Max \leq 29.56$
- $0.0 \leq Mean \leq 0.21$
- $0.73 \leq SD \leq 2.73$
- $0.85 \leq Kurtosis \leq 21.88$
- $-0.54 \leq Skwenes \leq 2.18$

The behaviour of assets return is different from an asset to another. However, most of distributions are more peaked than a Gaussian distribution (the Skewness $\neq 0$). The SP 500 index could being considered as normally distributed (Skewness=-0.15 , Kurtosis=0.99).

Table (4) is devoted to the Correlation Matrix:

Table 3: USA Historical Statistics

	Abbreviation	Min	Mean	SD	Skewness	Kurtosis	Max
SP500	S&P	-2.50	0.06	0.73	-0.15	0.99	2.54
Apple	App	-12.37	0.07	1.62	-0.54	6.81	8.20
Google	Goo	-8.09	0.09	1.34	1.28	16.27	13.86
Microsoft	Mft	-11.40	0.06	1.44	-0.23	11.57	10.40
Exxon	Exx	-4.18	0.00	0.94	-0.11	1.57	3.22
Berkshire	Ber	-0.90	0.03	0.42	-1.3	14.29	9.34
Wellsfargo	Wel	-4.98	0.08	1.04	0.07	1.92	4.37
Johnson	Joh	-2.88	0.05	0.84	-0.22	0.85	2.57
General.Electric	Gel	-5.35	0.04	1.07	0.33	4.74	7.34
JP.Morgan	JPM	-5.49	0.08	1.24	-0.07	1.49	5.48
Facebook	Fbk	-12.12	0.17	2.73	2.18	21.88	29.56
Wallmart	Wal	-4.36	0.01	0.89	-0.23	2.81	4.72
Procter.and.Gramble	PaG	-6.44	0.04	0.88	-0.20	5.68	4.01
Pfizer	Pfi	-4.41	0.06	0.97	0.00	1.58	4.23
Amazon	Ama	-11.10	0.10	1.87	0.54	11.98	14.11
Walt.Disney	WDi	-8.21	0.12	1.22	0.42	10.48	9.89
Coca.Cola	CCO	-5.96	0.01	0.92	-0.08	4.65	5.44
Gilead.sciences	GiS	-14.34	0.21	1.89	-0.06	8.52	13.72
Visa	Vis	-7.14	0.11	1.27	0.59	7.33	10.29
Citi.group	CGr	-6.22	0.10	1.48	0.16	1.50	6.42

Table 4: USA Correlation Matrix

	App	Goo	Mft	Exx	Ber	Wel	Joh	Gel	JPM	Fbk	Wall	PaG	Pfi	Ama	WDi	CCo	Gis	Vis	CGr
App	1.00	0.23	0.21	0.19	0.01	0.27	0.16	0.01	0.22	0.16	0.20	0.16	0.02	0.16	0.21	0.16	0.16	0.04	0.20
Goo	0.23	1.00	0.35	0.32	-0.03	0.40	0.33	0.08	0.36	0.24	0.25	0.26	0.10	0.49	0.36	0.25	0.28	0.09	0.39
Mft	0.21	0.35	1.00	0.34	0.00	0.37	0.29	0.06	0.36	0.12	0.27	0.27	0.05	0.29	0.30	0.27	0.18	0.12	0.33
Exx	0.19	0.32	0.34	1.00	0.06	0.56	0.48	-0.02	0.51	0.11	0.29	0.40	0.02	0.30	0.40	0.38	0.29	0.08	0.48
Ber	0.01	-0.03	0.00	0.06	1.00	0.04	-0.02	-0.04	-0.01	0.01	0.00	-0.03	0.01	-0.01	0.03	-0.00	0.09	0.05	0.02
Wel	0.27	0.40	0.37	0.56	0.04	1.00	0.53	0.00	0.69	0.16	0.36	0.34	0.02	0.35	0.54	0.38	0.34	0.07	0.66
Joh	0.16	0.33	0.29	0.48	-0.02	0.53	1.00	-0.00	0.44	0.15	0.41	0.50	0.01	0.29	0.43	0.44	0.37	-0.01	0.43
Gel	0.01	0.08	0.06	-0.02	-0.04	0.00	-0.00	1.00	0.01	-0.04	-0.01	-0.04	0.41	0.03	-0.03	-0.04	-0.02	0.36	0.01
JPM	0.22	0.36	0.36	0.51	-0.01	0.69	0.44	0.01	1.00	0.19	0.31	0.32	0.02	0.32	0.44	0.34	0.29	0.06	0.77
Fbk	0.16	0.24	0.12	0.11	0.01	0.16	0.15	-0.04	0.19	1.00	0.05	0.06	-0.04	0.22	0.21	0.07	0.24	-0.02	0.17
Wal	0.20	0.25	0.27	0.29	0.00	0.36	0.41	-0.01	0.31	0.05	1.00	0.43	0.02	0.20	0.41	0.38	0.20	0.02	0.31
PaGr	0.16	0.26	0.27	0.40	-0.03	0.34	0.50	-0.04	0.32	0.06	0.43	1.00	-0.05	0.17	0.34	0.49	0.26	-0.04	0.32
Pfi	0.02	0.10	0.05	0.02	0.01	0.02	0.01	0.41	0.02	-0.04	0.02	-0.05	1.00	0.04	0.01	-0.03	0.06	0.39	0.01
Ama	0.16	0.49	0.29	0.30	-0.01	0.35	0.29	0.03	0.32	0.22	0.20	0.17	0.04	1.00	0.36	0.20	0.27	0.09	0.33
WDi	0.21	0.36	0.30	0.40	0.03	0.54	0.43	-0.03	0.44	0.21	0.41	0.34	0.01	0.36	1.00	0.39	0.26	0.07	0.42
CCo	0.16	0.25	0.27	0.38	-0.00	0.38	0.44	-0.04	0.34	0.07	0.38	0.49	-0.03	0.20	0.39	1.00	0.19	0.01	0.34
Gis	0.16	0.28	0.18	0.29	0.09	0.34	0.37	-0.02	0.29	0.24	0.20	0.26	0.06	0.27	0.26	0.19	1.00	0.07	0.32
Vis	0.04	0.09	0.12	0.08	0.05	0.07	-0.01	0.36	0.06	-0.02	0.02	-0.04	0.39	0.09	0.07	0.01	0.07	1.00	0.06
CGr	0.20	0.39	0.33	0.48	0.02	0.66	0.43	0.01	0.77	0.17	0.31	0.32	0.01	0.33	0.42	0.34	0.32	0.06	1.00

In the American market, the correlation between two stocks is larger when they are from similar sectors, for example Facebook, Google, Amazon and JP Morgan, City Group and Johnson are positively correlated. There is no significant negative correlation between the assets return. All the correlations are low or medium.

3.3. Portfolio Optimisation

Using the data of the two markets, we will use the Conditional Mean-Variance and the Conditional Mean-Semivariance models to get optimal portfolios that we can invest in each market. The idea is to anticipate the future but putting into consideration the past. In the classical methods, Mean, Variance and Semivariance does not take account of the forthcoming data.

Our methodology consists in dividing the data into two samples: one for making the optimization (optimization sample) and the other for testing the efficiency of the methods (test sample). The optimization sample is used to determine the optimal weights for each method (classical Mean-Variance, Conditional Mean-Variance, Classical Mean-Semivariance and Conditional Mean-Semivariance). These weights are used for computing the optimal portfolio returns for each method.

In order to measure the performance of the proposed methods we use the sample test. The optimal portfolio returns are compared to the naive one and they are also used to assess performance against the HangSeng index and S&P 500 one. The following parameters and considerations will be used throughout this section:

- Optimal portfolio is determined for one period,
- For the target return E^*, many values are tested. We have decided to present only results with $E^* = 0.075\%$. Results with other target returns are similar.
- The benchmark is chosen equal to 0 ($B = 0$). This choice makes easy certain calculations.

- The kernel $\mathcal{K}$ is the multivariate Gaussian density,

$$\mathcal{K}(x) = \frac{1}{(2\pi)^{n/2}} \exp\left(-\frac{x_1^2 + \cdots + x_m^2}{2}\right),$$

- The bandwidths h are chosen by cross validation method and depend on each asset return observations (see previous sections). This choice is motivated by its popularity in nonparametric literature (see Arlot and Celisse (2010)).

- There is no transaction cost,

- Short selling is allowed.

From now on, the following abbreviation will be used:

- M-V= Classical Mean-Varince Model
- C.M-V= Conditional Mean-Varince Model
- M-DSR= Classical Mean-Semivarince Model
- C.M-DSR= Conditional Mean-Semivarince Model

To test our methods, we used the following procedure:

- to determine the optimal portfolio, we use all the data collected until May, 31th, 2015,

- returns collected from June, the 1st, 2015 until July, the 7th, 2015 are used to compare naive portfolio return (all the weights are equal to $\omega_i = \frac{1}{m}$) to those obtained by the other methods.

For the Chinese Market, results are summarized in the Figure (1). It is clear that, in term of returns, all the portfolios obtained from the optimization methods are dominating the naive one. The conditional methods perform better than the non conditional ones.

Figure 1: **Chinese Portfolio return (%)**

From Figure (1), we can extract the following comparative Table (5) which could inform us about the efficiency of each method compared to effective return obtained by the naive portfolio ($\omega_j = 1/m$), computed day after day starting from June, the 1st, 2015.

Table 5: Naive versus the other portfolio optimization methods in the Chinese Market

	Naive	M-V.	C.M-V	M-DSR	C.M-DSR
Naive	26/26	16/26	20/26	21/26	**22/26**

In the above table, the numerator of the fraction is the number of days on which methods perform better than the naive one. Here we see the supremacy of all the methods compared to the naive one. The conditional mean-semivariance method seems perform better than all the other methods.

In the same spirit and under the same conditions, the portfolio optimization processes, in the American Market, are summarized in Figure (2).

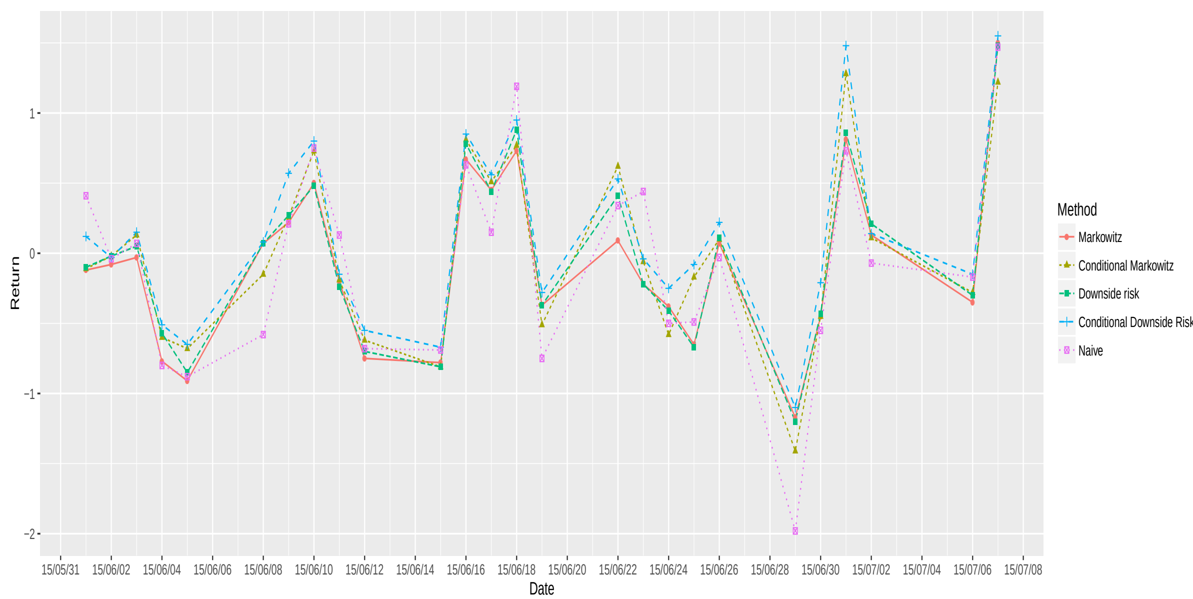

Figure 2: **American Portfolio return (%)**

On this market, we observe that the same finding continues to occur as in the Chinese Market. All conclusions, concerning the performance of the optimisation methods versus the naive one, are summarized in the following comparative table (Table 6). The conditionam methods perform better than the classical one.

Table 6: Naive versus the other portfolio optimization methods in the American Market

	Naive	M-V.	C.M-V.	M-DSR	C.M-DSR
Naive	26/26	13/26	17/26	16/26	**22/26**

3.4. Another Tool to Test The Efficiency of the Proposed Methods

The HangSeng and the S&P 500 indices will contribute to test the efficiency of the classical and the new portfolio optimization methods. We will compare the daily portfolio return obtained by the weights which are solutions of different optimization programs using all the data until May the 31th , 2015. From June the 1st, 2015 until July the 7th, 2015, we compute the portfolio returns and we compare them against the daily HangSeng and the S&P 500 indices. Results are exhibited in figure 4.1 (the Chinese Market) and figure 4.2 (the American Market).

- Red color signifies a negative difference between portfolio return and HangSeng Index (respectively S&P 500 index).
- Blue color signifies a positive difference between portfolio return and HangSeng Index (respectively S&P 500 index).
- Overall performance is the sum of all the differences during the test period.

We note that the comparison is in favour of the Conditional-Mean-Variance and the Conditional-Mean-Semivariance optimization methods. The overall performance varies depending on the method of portfolio optimization and markets:

- From 0.94% (Classical Markowitz method) to 12.64% (Conditional Mean-Semivariance method) for the the Chinese Market,
- From 0.09% (Classical Markowitz method) to 1.42% (Conditional Mean-Semivariance method) for the American Market.

These mouthwatering results deserve more practice and more tests over a long period of time and on different markets using different financial products.

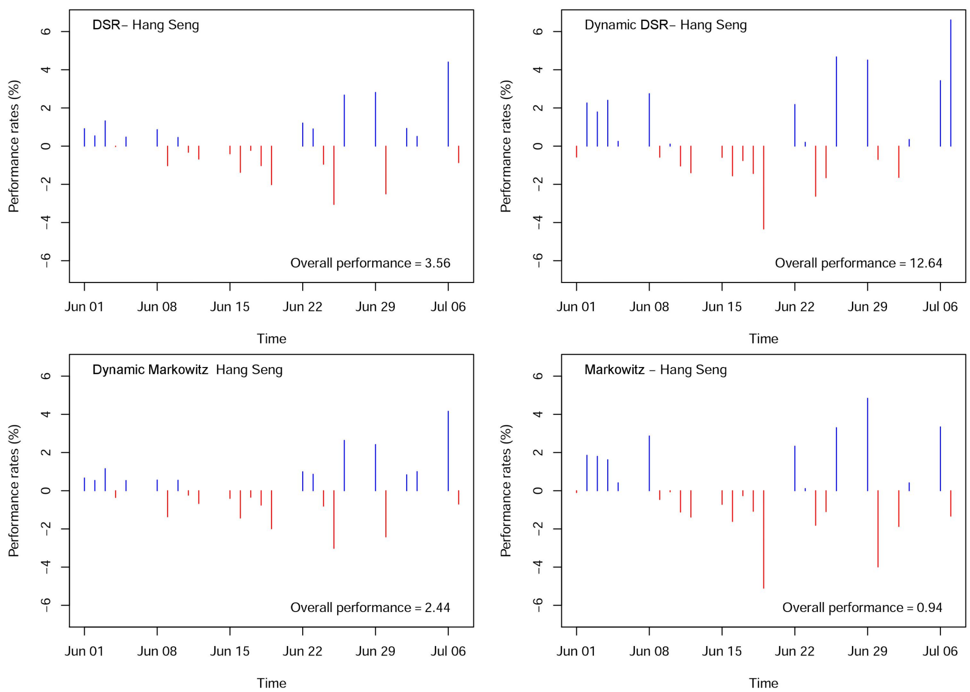

Figure 3: **HangSeng index versus return of optimal portfolio**

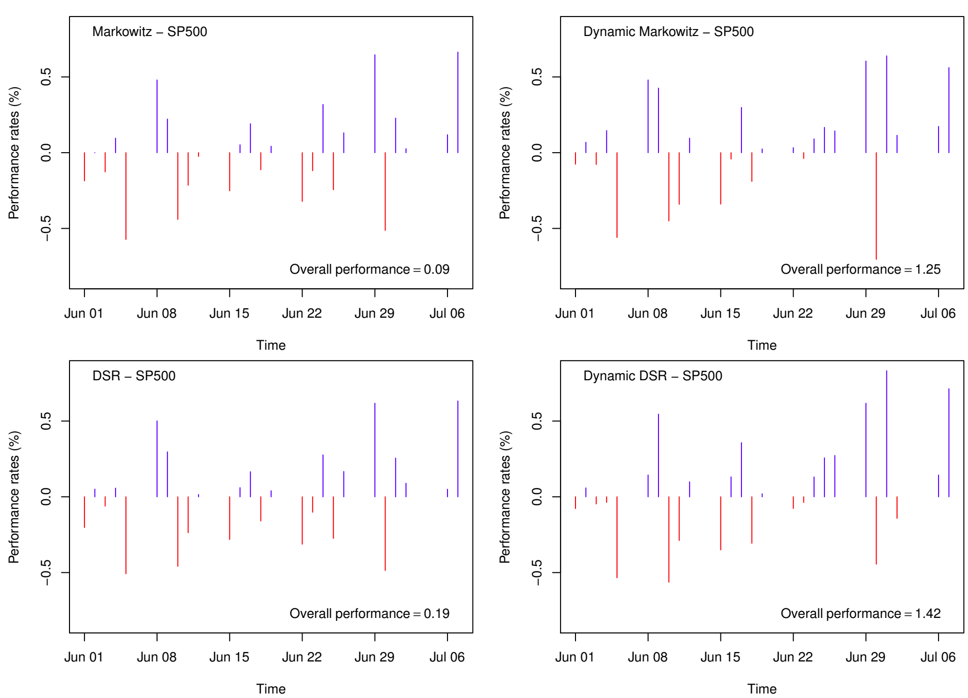

Figure 4: **S&P500 index versus return of optimal portfolio**

4. Conclusion

In this paper, we developed two new approaches in order to get an optimal portfolio minimizing two conditional risks.

- The first risk is based on conditional variance. In this case, the optimization method is an extension of the classical Mean-Variance model. What we proposed is to replacing Mean and Variance by Conditional Mean and Conditional Variance estimators.

- The second one is based on Conditional Semivariance. In this case, the optimization method is an extension of the classical Mean-Semivariance model. What we proposed is to replacing Mean and Semivariance by Conditional Mean and Conditional semivariance estimators.

This novelty, using conditional risk, gives a new approach and a more efficient alternative to get an optimal portfolio. In fact, all the other methods do not anticipate the future and just extrapolate to the future what we observed in the past.

In both cases, the optimization algorithm involved using the Lagrangian method. Even, our results seem interesting, the efficiency of our methods should be confirmed on other markets and with other various assets. Kernel methods, belonging to nonparametric methods, are used to estimate Conditional Mean, Variance and Semivariance. Product Gaussian densities are used as kernel. It will be very helpful to choose typical multivariate kernels. Similarly, we should develop a global method to choose the bandwidth which is crucial in nonparametic estimation.

Back to results of this paper: the Conditional Mean-Semivariance seem most appropriate to get an optimal portfolio using the data of the Chinese and the American markets. By the way, Conditional Mean-Variance is more efficient than Mean-Variance method. Similarly, the Conditional Mean-Semivariance is better than the Mean-Semivariance method.

Thanks to its robustness, it is also reasonable to substitute the conditional median to the conditional mean and to propose an optimization model based on Conditional Median and conditional variance or conditional Median and Conditional Semivariance. This topic will be treated as a matter for future research.

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